TYPE 713: RADIANT SLAB WITH EMBEDDED PIPES (INTERFACES WITH ZONE AIR TEMPERATURE)

General Description

This component is intended to model a radiant floor-heating slab, embedded in soil, and containing a number of fluid filled pipes. The heat transfer within the slab is assumed to be conductive only and moisture effects are not accounted for in the model. The model relies on a three-dimensional finite difference method, solving the resulting inter-dependent differential equations using an iterative approach. The user may define any number of pipes within the slab through An external data file containing information about the path that the pipes follow through the slab for each pipe. The slab is assumed to be embedded in soil and the user may define bottom and/or perimeter insulation that extends below the slab if desired. The slab may also be exposed to incident radiation whether from interior lights or from the sun. Default numbers of maximum slab nodes, maximum pipe nodes and maximum number of pipes are set, but may be increased by modification of the Fortran code. This model differs from Type706 in that it does not extend the 3D finite element grid into the soil that surrounds the slab. Instead, the zone temperature communicates (through the slab) with a ground surface temperature generated by the Kasuda [1] correlation.

If you wish to use this radiant slab model in conjunction with Type56, you should use Type712 instead. This version (Type713) is designed to be used with building models that do not contain the notion of a boundary wall and compute interior surface temperatures. For more detail on this distinction, please see section 1 of the detailed description below.

Nomenclature

m	[kg]	Mass.
C_p	[kJ/kg.K]	Specific heat.
$\dot{Q}_{in}$	[kJ/hr]	The rate at which energy enters a fluid, soil or slab node.
$\dot{Q}_{out}$	[kJ/hr]	The rate at which energy exits a fluid, soil or slab node.
$\dot{m}$	[kg/hr]	Mass flow rate of fluid entering a fluid node.
$T_{fluid,i}$	[ºC]	The temperature of fluid in a given fluid node.
$T_{x,y,z}$	[ºC]	The temperature of a given soil or slab node.
x	[m]	The distance along the length direction.
y	[m]	The distance along the width direction.
R_{inside}	[hr.m^2.K/kJ]	The thermal resistance between the fluid and the pipe.
R_{pipe}	[hr.m^2.K/kJ]	The thermal resistance of the pipe material.
$R_{contact}$	[hr.m^2.K/kJ]	The thermal contact resistance between the pipe and surrounding soil or slab.
k	[kJ/m.K]	Thermal conductivity.
dx	[m]	The x direction dimension of a soil or slab node.
dy	[m]	The y direction dimension of a soil or slab node.
$T_{ambient}$	[ºC]	The temperature of a node's surroundings. For an indoor node, $T_{ambient}$ is the temperature

		of the zone. For an outdoor node, $T_{ambient}$ is the outdoor dry bulb temperature.
$T_{surface}$	[ºC]	The surface temperature of a node exposed to ambient conditions.
$T_{surroundings}$	[ºC]	The effective temperature with which a node radiatively transfers energy. For an indoor node, $T_{surroundings}$ is the surface temperature of zone walls. For an outdoor node, $T_{surroundings}$ is the effective sky temperature.
$h_{convective}$	[ºC]	The convective heat transfer coefficient for the surface of a node.
σ	[$W/m^2.K^4$]	The Stefan-Boltzmann constant ($5.67x10^{-8}$ $W/m^2.K^4$).
ε	[0..1]	The emissivity of a surface.
$Q_{ambient}$	[kJ/hr]	Additional radiation incident on a node.

Detailed Description

1. Appropriate Building Models

Simplified building models such as the lumped capacitance or degree day models (for example standard Types 80, and 12 or TESS Type660) do not contain the concept of a boundary wall. Defining a wall as a boundary wall assumes that there is either a known temperature on the other (non-zone) side, or that the temperature on the non-zone side is identical to the temperature in the zone. In both cases, the temperature specified is typically the surface temperature at the interface between the non-zone side of the wall and its surroundings, as shown in Figure 1.

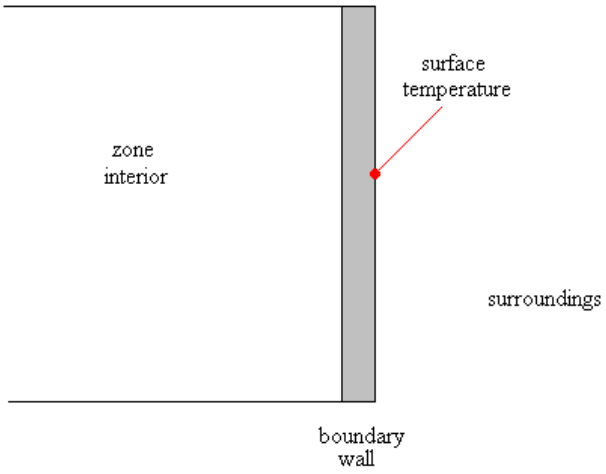


Figure 1: Boundary Wall Definition

This scheme allows the boundary wall's capacitance to effect the simulation of the zone's thermal behavior. In order to include an external model of a slab (in other words a model of a wall that is not part of the model of the building) an iterative scheme must be included so that the capacitance effects are neither lost, not accounted for twice. Type713 does NOT include such a scheme. Type712 does and should be used in conjunction with more complex building models (such as Type56). Type713 is, however appropriate for use with simplified building models. Instead of computing the surface temperature shown in Figure 1, Type713 contains algorithms that allow for the calculation of a convective and radiative energy balance between the zone surface of the wall (or slab) and the zone air temperature. These energy balances are addressed in the following sections.

2. Slab and Surroundings

Type713 divides the slab and its surroundings into two parts. First is the slab itself, for which the user specifies length, width, thickness, thermal properties, and the number of nodes in the x and y directions as shown in Figure 2.

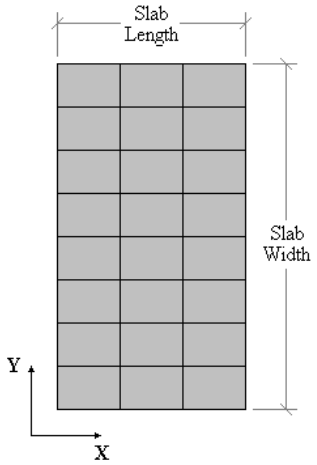


Figure 2: Slab Dimensions and Coordinate System

The slab is embedded in (surrounded on its sides and back by) soil referred to as the far-field, which is assumed to be an infinite energy sink/source. In other words, energy transfer with the far-field does not result in a temperature change of the far-field.

The sizes of the slab nodes are automatically determined by the length and width of the slab and by the user specified number of nodes along the length and along the width of the slab. The slab is assumed to have a single node in the z direction.

3. Pipe Layout

Type713 allows users flexibility in describing how heating and/or cooling fluid is passed through the slab. Users are first asked to specify (as TRNSYS input file parameters) the number of pipes that pass through the slab and the number of fluid nodes into which the pipe should be broken. The isothermal nodes must be specified from the start of the pipe to the end of the pipe along the direction of flow. The greater the number of nodes, the greater the accuracy but at the cost of simulation speed. Users may specify multiple pipe nodes per slab node but by definition a pipe node cannot extend over multiple slab nodes. For each pipe, the user must provide an external data file that describes that pipe's path through the slab (examples are given below). The data file must contains the fraction of pipe length assigned to that pipe node and the coordinates of the slab node in which the pipe is embedded. Figure 3 shows an example in which a twelve-node pipe (uniform pipe nodal lengths in this example) passes through a twelve-node slab. In this example, the boundaries between pipe nodes correspond to the slab node boundaries.

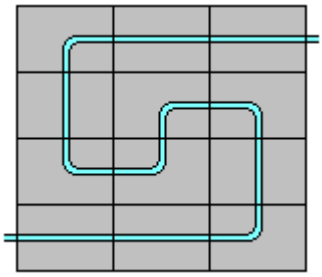


Figure 3: Example Pipe Layout with One Pipe Node Per Slab Node

The data file describing this pipe path is shown in Figure 4. The data file contains as many rows as there are slab nodes in the pipe. In each row, the first value is the fraction of the total pipe length assigned to the pipe node followed by the x, and y coordinates of the slab node containing the pipe node.

0.0833	1,1
0.0833	2,1
0.0833	3,1
0.0833	3,2
0.0833	3,3
0.0833	2,3
0.0833	2,2
0.0833	1,2
0.0833	1,3
0.0833	1,4
0.0833	2,4
0.0833	3,4

Figure 4: Data File Describing Figure 3 Pipe Path

Figure 5 shows a slightly more complex example in which two pipes pass through a single slab. This second example is meant to illustrate the method for specifying multiple pipe nodes per slab node (the perimeter pipe in Figure 5) and the method for defining different lengths of pipe contained in a slab node (the interior pipe in Figure 5).

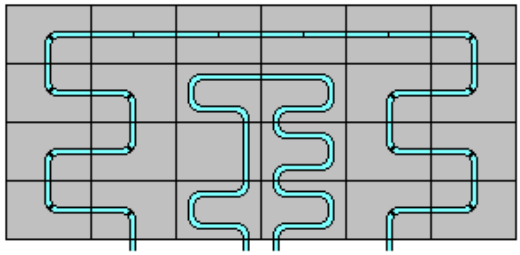


Figure 5: Slab Containing Two Pipe Paths

Data File Describing Flow Path for Perimeter Pipe	
0.0277	5,1
0.0277	5,1
0.0277	6,1
0.0277	6,1

Data File Describing Flow Path for Interior Pipe	
0.182	3,1
0.091	3,2
0.182	3,3
0.182	4,3

0.0277	6,2
0.0277	6,2
0.0277	5,2
0.0277	5,2
0.0277	5,3
0.0277	5,3
0.0277	6,3
0.0277	6,3
0.0277	6,4
0.0277	6,4
0.0277	5,4
0.0277	5,4
0.0277	4,4
0.0277	4,4
0.0277	3,4
0.0277	3,4
0.0277	2,4
0.0277	2,4
0.0277	1,4
0.0277	1,4
0.0277	1,3
0.0277	1,3
0.0277	2,3
0.0277	2,3
0.0277	2,2
0.0277	2,2
0.0277	1,2
0.0277	1,2
0.0277	1,1
0.0277	1,1
0.0277	2,1
0.0277	2,1

0.182	4,2
0.182	4,1

Figure 6: Data Files for Figure 5 Slab

It is important to note that while the user may specify there to be more than one pipe in a given slab node, these pipes will not "talk" to each other directly due to a temperature difference between them. They will each affect the same slab node, however, and so affect each other indirectly.

4. Energy Balances

The basic energy balance on a fluid node or node contained in the slab is shown in Equation 713.1:

$$mC_p \frac{dT}{dt} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \text{(Eq. 713.1)}$$

The equations describing Q_{in} and Q_{out} change depending upon the type of node and its location. Four types of nodes exist: fluid nodes, boundary nodes (between the slab and the far-field for example), and surface nodes.

4.1 Fluid Nodes

It is assumed that energy enters and leaves a fluid node through fluid flow into that section of pipe and through heat transfer with the slab. Conduction due to a temperature difference between fluid nodes is not considered. Thus when flow ceases, the fluid tends toward a steady state temperature only through conduction with the surrounding slab nodes and not through fluid conduction along its length. The appropriate Q_{in} term for flow is then:

$$\dot{Q}_{in} = \dot{m} C_p \left(T_{fluid,i-1} - T_{fluid,i} \right) \quad \text{(Eq. 713.2)}$$

Energy is transferred from the pipe node to the surrounding soil/slab node via conduction according to Equation 713.3 where $\overline{T}$ denotes the average slab temperature over the time step.

$$\dot{Q}_{out} = UA\left(T_{fluid,i} - \overline{T}_{x,y} \right) \quad \text{(Eq. 713.3)}$$

It should be pointed out that the heat transfer out of the pipe node is then by definition the heat transfer into the slab node.

The UA term in equation 713.3 is made up of three parts; an internal convection resistance from the fluid to the pipe, a pipe wall resistance, and a conductive resistance in the slab.

The internal heat transfer resistance is simply:

$$R_{in} = \frac{1}{h_{inside} A_{surface,inner}} \quad \text{(Eq. 713.4)}$$

where:

$$h_{inside} = \frac{Nusselt\ k}{d_{pipe,i}} \quad \text{(Eq. 713.5)}$$

and for laminar flow (Reynolds number < 2300):

$$Nusselt = 4.36 \quad \text{(Eq. 713.6)}$$

and for turbulent flow (Reynolds number ≥ 2300):

$$Nusselt = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{(1/3)} \quad \text{(Eq. 713.7)}$$

The Reynolds number and Prandtl numbers are based on the inside diameter of the pipe and the fluid properties. The pipe wall resistance is defined by the radial conduction equation:

$$R_{wall} = \frac{\ln\left(\frac{d_{pipe,o}}{d_{pipe,i}} \right)}{2\,\Pi\,k\,l} \quad \text{(Eq. 713.8)}$$

In order to determine the conduction resistance from the pipe outer surface to the slab node, the conduction path length must be specified. In the case of a radiant floor, it is difficult to know this length since the exact path that the pipe follows through the slab node is not known. Instead, a contact resistance ($R_{contact}$) that describes the conductance between the pipe and the surrounding slab node is requested of the user as a parameter. This parameter can be thought of as a tuning parameter; used to adjust the heat transfer to measured data (if available).

The expression for the overall U value is:

$$U = \frac{1}{R_{inside} + R_{pipe} + R_{contact}} \quad \text{(Eq. 713.9)}$$

4.2 Internal Slab/Soil Nodes

If a node is contained within the near field and does not have any pipes passing through it, its energy balance is made up of purely conductive terms. The equations for Q_{in} and Q_{out} are:

$$\dot{Q}_{in} = \frac{k_{soil}\, dy\, dz}{x_i - x_{i-1}}\left(\overline{T}_{x-1,y} - T_{x,y}\right) + \frac{k_{soil}\, dx\, dz}{y_i - y_{i-1}}\left(\overline{T}_{x,y-1} - T_{x,y}\right) \quad \text{(Eq. 713.10)}$$

$$\dot{Q}_{out} = \frac{k_{soil}\, dy\, dz}{x_{i+1} - x_i}\left(T_{x,y} - \overline{T}_{x+1,y}\right) + \frac{k_{soil}\, dx\, dz}{y_{i+1} - y_i}\left(T_{x,y} - \overline{T}_{x,y+1}\right) \quad \text{(Eq. 713.11)}$$

It should be noted that some internal soil nodes may have an additional resistance term added because Type713 allows insulation on the perimeter of the slab to extend down below the slab as far as desired.

4.3 Boundary Nodes

Nodes in the slab on the boundary with the far-field are governed purely by conduction as long as they do not also contain fluid pipes. The-far field temperature to which the node conducts is not affected by energy passing from the node to the far-field, only by time of year as given by the Kasuda correlation. These nodes may have an additional pure resistance built into their energy balance. The resistive term is used to account for insulation around the perimeter of or beneath the slab.

In the examples above, the slab thermal conductivity and soil thermal conductivity are assumed equal. If this is not the case (which it usually isn't), the $\dot{Q}_{in}$ and $\dot{Q}_{out}$ terms are slightly modified to account for the change in thermal conductivity due to the two materials. These algorithm modifications are built into the model but are not detailed in the documentation. Refer to the source code for the governing equations if further clarification is needed.

The conduction length between the nodes in the slab and the far-field is one-half of the corresponding nodal distance. For example, the conduction length between a node in the slab and the far-field is dz/2.

4.4 Surface Nodes

Surface nodes are handled in a slightly different manner from other nodes because they interact with the surroundings, or with a known surface condition. For surface mode 1, the surface temperature is calculated from an energy balance on the surface (assuming the surface has negligible mass). The energy balance on the surface reveals that the energy conducted to the surface by the node must equal the energy convected (to the ambient) and radiated (to the surroundings) from the surface minus any

incident solar radiation (or incident radiation from lighting etc.). The energy balance on the surface is shown schematically in Figure 7.

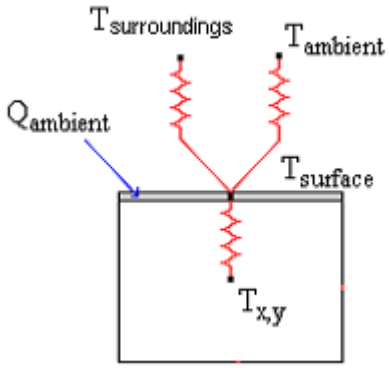


Figure 7: Energy Balance on a Surface Node

The energy transferred between from the surface to the environment contains terms for both radiative heat transfer between the surface and surrounding temperature and a standard convection term between the surface and the ambient. The radiation term may be linearized with ambient temperature using Equation 706.12.

$$h_{radiation} = \left(\varepsilon\sigma\left(T_{ambient} + T_{surroundings}\right)\left(T^2_{ambient} + T^2_{surroundings}\right)\right) \qquad \text{(Eq. 713.12)}$$

In the outdoor case, $T_{surroundings}$ is the effective sky temperature and $T_{ambient}$ is the outdoor dry bulb temperature. In the indoor case, $T_{surroundings}$ is the effective temperature of the zone walls and $T_{ambient}$ is the zone dry bulb temperature. The user can choose to neglect the radiation exchange calculations by setting either the slab or the soil emmisivity and absorptance to zero.

The energy balance on the surface is then:

$$0 = \left(h_{radiation} + h_{convection}\right)\left(T_{surface} - T_{ambient}\right) - Q_{ambient} - \frac{k\,dx\,dy\left(T_{x,y} - T_{surface}\right)}{dz/2} \qquad \text{(Eq. 713.13)}$$

This energy balance is solved for the surface temperature which is then used for the conduction equations for the slab node. Surface mode 2 uses the Kasuda correlation to set the surface temperature while surface mode 3 relies on an input surface temperature to the model.

The resistance network is shown in Figure 8 for a surface node. For purposes of clarity, conduction in the y direction has been neglected from the figure. The conduction length for heat transfer to/from the surface is half the nodal height (dz/2).

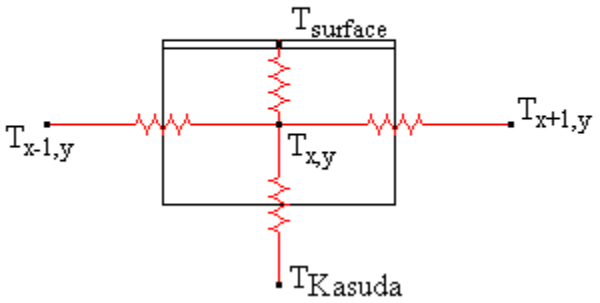


Figure 8: Energy Balance on a Surface Node

5. Solving the Resulting Equations

In this type of system, energy is transferred from a fluid in a pipe into an adjacent slab node and then this heat is conducted to other the far field or is convected / radiated to the surroundings. The problem breaks down into the required solution of two coupled differential equations of the form of Equation 713.1; one for each slab node and one for each pipe node. The equations are coupled in that the pipe nodal equations depend on the pipe fluid temperatures and on the slab nodal temperatures. While there are numerous methods available to solve coupled differential equations, it was decided to solve the problem with an approximate analytical solution. The analytical solution has several inherent advantages over numerical solutions. First, the subroutine solves its own mathematical problem and does not rely on non-standard numerical recipes that must be attached to the subroutine. In this way, the subroutine can be imported into any FORTRAN compiler, whereas numerical recipes depend upon external libraries that may or may not exist for one compiler or another. Secondly, some of the other solution methods (mainly the numerical solutions) are dependent on the simulation time step and may not converge under certain circumstances commonly encountered in these types of systems (high fluid flow rates for example). The analytical solution is time step independent but does require an iterative solution inside the subroutine to solve the coupled differential equations. While solving two coupled differential equations iteratively can sometimes lead to convergence problems, this has not been found to be the case in using this set of components.

To solve the differential equations analytically, each of the nodal the equations are placed into the form:

$$\frac{dT}{dt} = aT + b \qquad \text{(Eq. 713.14)}$$

where T is the dependent variable (temperature in this case), t is time, a is a constant and b may be a function of time or the dependent variable. If b is a constant, the solution of this differential equation can be readily solved by a standard TRNSYS utility routine called DIFFERENTAL_EQUATION. If b is not constant, then a reasonable approximation to the analytical solution can be found by assuming that b is constant over the time step and equal to its average value over the time step.

At any time (for **a** not equal to zero):

$$T_{final} = (T_{initial} + \frac{\overline{b}}{a})\ e^{a\Delta t} - \frac{\overline{b}}{a} \qquad \text{(Eq. 713.15)}$$

where

$$\overline{b} = b(\overline{T}) \qquad \text{(Eq. 713.16)}$$

and

$$\overline{T} = \frac{1}{a\,\Delta t}\left(T_{initial} + \frac{\overline{b}}{a}\right)\left(e^{a\,\Delta t} - 1\right) - \frac{\overline{b}}{a} \qquad \text{(Eq. 713.17)}$$

With this assumption, the problem becomes straightforward to solve. Simply write the differential equation for each node in the correct form, determine a and $\overline{b}$ for each node and solve for T_{final} and T_{ave}. Then recalculate $\overline{b}$ for each node and iterate until all temperatures converge.

While the assumption that b is constant over the time step (and equal to its average value) is not technically correct in our case (b for the pipe node is a function of the slab temperature for example), it is a reasonable approximation for the small time steps we are using in the TRNSYS simulation (maximum suggested time step = 1 hour). The b terms contain the temperatures of other slab nodes and pipe nodes. These temperatures in the b term are assumed to be constant for the solution of the nodal differential equations at their average value over the time step.

References

[1] Kasuda, T., and Archenbach, P.R. "Earth Temperature and Thermal Diffusivity at Selected Stations in the United States", ASHRAE Transactions, Vol. 71, Part 1, 1965